discharge in gases; studies of the mysterious photo-electric effects at the surface of bodies and throughout the body of a gas, and that cloud-producing action which seems to provide us with the opportunity of isolating electrons, obtaining a drop of liquid with only one of these upon it; studies of kathode rays and of the velocity of the electric carriers in them. The argument by which, in this concluding study, there is an attempt to prove that these cannot be either larger than molecules, nor of a comparable size, is not particularly conclusive. Among the reasons given for concluding that these carriers are not larger, it is assumed that the resistance to their motion would obey the laws of motion of a body moving slowly through a viscous medium. This is not at all true of a minute projectile moving with a velocity of at least one hundred times the velocity of the molecules of the gas. To such a projectile the gaseous molecules would be practically at rest, and the resistance would be, as Lenard observed, proportional to the density of the gas. The argument against their being of a size about the same as that of the other gaseous molecules depends on the assumption that the effective size of a molecule is independent of its rate of motion, a conclusion which is at variance with the known laws of diffusion of gases at various temperatures, and is contrary to such a probable hypothesis as that the actions between molecules is more like that between centres of force with rapidly changing forces near them than like that between hard spheres. For a comet to be deflected by a given amount when going near the sun, it should pass much closer to the sun if its velocity is large than if its velocity is small. The effective size of the sun as a deflecting cause for comets very largely depends on the velocity of the comet. On these principles rapidly moving molecules might go much further through a gas than the free path of an ordinary molecule. When the disagreement between the theory of hard spheres and the observed laws of diffusion at different temperatures is measurable, and it only corresponds to alterations of velocity to about twice the normal, we may naturally expect great variations when dealing with velocities of about one hundred times the normal, which would correspond to a temperature of some 2,000,000° C.

At the same time, there is a good deal of other evidence in favour of the suggestion that in kathode rays we are dealing with carriers of smaller mass than atoms. One would naturally conclude that we were here dealing with what might be described as disembodied electrons, *i.e.* with electrons separated from the atoms with which they seem in general so intimately connected. Most theories of electrolysis assume that an electron can be transferred from the atom in the liquid to the plate, and almost any theory of metallic conduction must involve the transference of electricity from molecule to molecule in the conducting solid. The only difference between these cases and that of the supposition that kathode rays are torrents of electrons is as to the distance the electron can travel from molecule to molecule. If it can ever leave one molecule to join on to another, why may it not jump some centimetres of space between them? This reconciles to some extent the radiant matter and pure ether theories of the kathode rays. It makes them essentially an ether phenomenon, while at the same time they are streams of particles: it makes them light on the emission theory.

It ought to be possible to test whether these emissions are material or not, by collecting them in a small vacuum tube. It is not generally appreciated how very small a mass of gas there is in a small highly exhausted vacuum tube, though it contains enormous numbers of molecules. At the pressure of a millionth of an atmosphere, which is quite a large pressure of about a dyne per sq. cm., there could be collected in a few minutes in a tube of several cubic millimetres capacity a quantity of hydrogen which it would require 100,000 years collecting at the same rate in order to collect a gramme of it.

In every way the book is suggestive, interesting, and inspiriting, and as such should fulfil its purpose of provoking research and advancing science.

GEOMETRY OF POSITION.

Lectures on the Geometry of Position. By Theodor Reye, Professor of Mathematics in the University of Strasburg. Translated and edited by Thomas F. Holgate, M.A., Ph.D., Professor of Applied Mathematics in North-western University. Part I. Pp. xix + 248. (London: Macmillan and Co., Ltd. New York: The Macmillan Company, 1898.)

AT last there is an English edition, translated in America, of the first part of Prof. Reye's "Geometrie der Lage," of which the first edition appeared as long ago as 1866. The methods followed offer so many advantages, and the style adopted is so lucid, that we heartily welcome this translation, and we with confidence express the hope that it will awaken new interest in the study of this charming subject in this country as well as in America.

The book owes its origin to lectures delivered to engineering students at the Polytechnic School at Zürich. They were undertaken with the view of giving the students that geometrical knowledge which they required in order to listen with advantage to Culmann's lectures on graphical statics.

Culmann based his new science upon the very abstract "Geometrie der Lage" by Von Standt, and this prescribed the course which Reye had to follow. He did so willingly, and says in the preface to the first edition:—

> "One principal object of geometrical study appears to me to be the exercise and the development of the power of imagination in the student, and I believe that this object is best attained in the way in which Von Standt proceeds."

The object of Reye's lectures, therefore, is not only to impart knowledge of conics and quadric surfaces, &c., but also and especially to educate the student's mind in the faculty of easily and readily realising geometrical figures in space, a faculty very important to engineers.

"Geometry of position," as opposed to "metric geometry," with which alone Euclid deals, excludes all measurement. But whilst the older authors and founders of this modern science based it on Euclidian propositions, Von Standt has shown that these can be altogether avoided, and Reye follows this course rigidly.

Here everything is based upon harmonic points in a range, and harmonic rays in a pencil, or, as the translator calls it, a sheaf.

All the previous knowledge needed consists of a clear notion of the concepts of point, line and plane, and their fundamental properties, such as the propositions that two points determine a line, two planes meet in a line, &c., together with a power of imagining these elements in their mutual relation in space; for instance, that a line may lie in a plane, or else can cut it in one point only.

These ideas being in the mind of the reader, nothing more is required, and they will themselves gradually become developed and gain in clearness as he proceeds.

It goes without saying that such a system requires a number of new concepts; namely, those of the primitive forms, "the elementary prime forms" in Leudesdorf's translation of Cremona's "Projective Geometry." These are explained in Lecture i.

In the second lecture the correlations between the prime forms as given by projections and sections are considered, and these lead to the ideal elements at an infinite distance. By their aid the exceptions due to parallelism are got rid of. At the same time a first instance is given of the leading principle of "correspondence" between the elements in two or more prime-forms.

In Lecture iii. the principle of duality appears as a natural and simple consequence of the first and fundamental properties of the elements.

After these preliminary lectures we come in the next to "harmonic forms," which are deduced by aid of certain simple constructions in three dimensions without reference to measurement. There is added a discussion of their metrical properties.

In the fifth lecture we have the projective properties of primitive forms, and the all-important principle of correspondence between the elements in two of them. It ends with the generation of curves of second order and second class, respectively, by projective ranges and sheaves (flat pencils), the actual construction of points on, or tangents to such curves being given.

With this the reader has finished the preliminary work. It may at first have appeared to him somewhat tedious, as he is unable to appreciate the usefulness and fertility of the definitions and methods explained. He has acquired many definitions and plenty of new ideas, but not as in Euclid at every step a tangible result. This is now all changed. The next chapter shows that the time and trouble expended are to be amply repaid, for now the study of the curves mentioned begins, and he is told that these curves are the conic sections. Side by side with these go the generation and the properties of cones of the second order, and cones of the second class, but these are at once dropped, as their properties are an immediate consequence of the curves. The latter are now investigated in detail. Their construction is more closely examined, and leads almost at once to the celebrated theorems of Pascal and Brianchon. It also follows that five points or tangents determine a curve of the second order or class.

The fact that these fundamental properties appear at the very beginning is characteristic of the whole method, and this shows that the method is a natural one.

In the seventh chapter the more immediate consequences of these theorems is considered. The curves are also classified as ellipses, hyperbolæ, and parabolæ, according to their elements at infinity.

In Chapter viii. the special case of Pascal's theorem when the hexagon is reduced to a quadrilateral leads to the theory of pole and polar, and these are in the next lecture specialised by aid of the lines and the points at infinity so as to give the properties of centres, diameters, and axes of conics. In this lecture, too, the equations of the conics are obtained, and thus the proof is given that the conics studied by aid of coordinate geometry are the same as those here considered.

Of course the principle of duality is constantly used, and properties of curves of second order and second class are treated simultaneously till in Lecture vii. their identity is proved. Numerous other theorems of conics are given at every stage, but we have only mentioned a few in order to show the sequence in which they follow.

Of this enough has been done, and the contents of the rest of the book can be dealt with more summarily.

Lecture x. treats of the ruled quadric surfaces; the next of projective relations of "elementary forms," viz. the range, the sheaves, curves and cones of second order, and the ruled quadric surfaces.

These relations lead to the "theory of involution," Lecture xii. The metric properties are considered in Lecture xiii., and applied to establish the *focal properties* of conics; whilst in Lecture xiv. problems of the second order are solved, and "imaginary elements" introduced in the manner of Von Standt. By their aid problems are solved by actual construction on the drawing board, in which among the given data are "imaginary" elements; for instance, the problem of drawing a conic which passes through three given points, and the two points in which a given line cuts a given conic in the case where these points have no actual existence.

It will be seen that hereby again the greatest generality is obtained, and that the exceptions which would occur it the different case where the line cuts the conic, touches it, or does not cut it, are to be treated separately.

In the fifteenth and last lecture, principal axes and planes of symmetry, focal axes, and cyclic planes of cones of the second order are dealt with.

This finishes the first part of Reye's work, which is restricted to the investigation of curves and surfaces generated by projective primitive forms of one dimension, the range of points, the sheaf of lines and the sheaf of planes, that is the range and the flat and axial pencil.

The projective and primitive forms of two dimensions give rise to quadric and cubic surfaces and twisted cubic curves, whilst the forms of three dimensions, viz. the space with points and planes as elements lead to the theory of co-linear and reciprocal spaces. These form the contents of Vol. ii. The translator does not say anything about it, but as the volume before us is marked both on the title-page and the back as Part i., there seems reason to hope that the translation of the second part will soon follow.

There is, however, an appendix to Part i., which has still to be mentioned. It contains an account of the principle of reciprocal radii, Dupin's cyclide, ruled surfaces of the third order, quadrangles and quadrilaterals, which are self-polar with respect to conic sections, and lastly a pretty full, though condensed, account of nets and webs of conic sections.

The translation is very well done. It has evidently been a labour of love to Prof. Holgate. He has, however, in the terminology departed somewhat from that which has been used in this country. He has altogether eschewed the word "pencil"; instead of it he uses for the flat and axial pencils "sheaf of rays" and "sheaf of planes," thus appropriating the word sheaf to one-dimensional forms, whilst it has in England been used for pencils of lines and planes in space. For these he takes the word "bundle," thus adopting the German phraseology. We think it is a pity he has made this change, not because the old terminology is better, but because it makes the adoption of a uniform system more difficult.

The translator has made a few changes from the original. He has broken up the lectures into articles which he has numbered, and he has placed the numerous exercises at the end of each lecture to which they belong, instead of leaving them together at the end of the book as in the original. He has also added an index. These are decided improvements. In his preface he gives a short account of the history of projective geometry.

As far as the paper, the printing and the figures are concerned, the book ranks among the best we have seen; in these respects it is, in fact, above praise.

We recommend the book to *all* students of mathematics, and advise them not only to read it, but also to actually make accurate drawings of at least a great number of figures; especially to construct a variety of conics from given conditions, and to solve as many as possible of the problems proposed. These constructions should be made accurately on the drawing board. They offer a wonderful incitement to accuracy and neatness, as every fault in drawing becomes at once evident to the eye. For this reason the schoolmaster who has to teach mathematics and geometrical drawing on the modern or science sides of schools, will find it of great advantage in his teaching. The book will afford him innumerable examples for geometrical drawing, even if he cannot much hope to enter at present on teaching the theory; although there is no reason, as far as the difficulty of the subject is concerned, why he should not introduce a good deal in the higher forms. It is most stimulating to young students, as is evidenced by the fact that students have repeatedly told us that, although at school they could not take any real interest in Euclid, and found geometry very hard, they found the theory of conics as treated according to Reye delightfully easy and interesting. In fact, the subject is full of life; the figures grow organically one out of the other, and the Propositions arrange themselves in a natural sequence. O. HENRICI.

RACIAL ANATOMY.

Observations sur les Variations Musculaires dans les Races Humaines. Par Théophile Chudzinski. Memoires de la Société d'Anthropologie de Paris, Ser. 3; t. ii.; fasc. 2. (Paris, 1898.)

THIS posthumous work of Chudzinski is a laborious and systematic attempt to open up quite a new field of anthropological investigation. It may be said, with perfect truth, that the study of the various races of mankind has been superficial and empirical; observations hitherto have been confined to features, colour of skin, size of body, proportion of limbs and bone measurements. With the exception of the late Prof. Giacomini of Turin, Chudzinski is practically the only man that ever went beneath the skin to discover the real and essential features that separate one race of man from another. There is no desire here to minimise the value of the few briefly recorded dissections, made on dark-skinned waifs that have died in Europe, and found scattered in the anatomical literature of the present century; the observations on the brains of negroid races by Parker, Broca, Gratiolet, Waldeyer, Marshall, Tiedemann, Calori and Barkow are good so far as they go, dissections of individual specimens made by Flower and Murie, Cuvier and Turner, are efforts in a right direction, but the merit of having opened up a systematic investigation of the anatomy of the races of mankind, taking a group of individuals to represent the race, remains to Chudzinski and Giacomini. This treatise is a record of the muscles of ten negroes, five negresses, five individuals belonging to yellow-skinned races, contrasted with the muscles of six individuals of a white-skinned race.

The result of this laborious investigation is to show that, on an average, in size, in proportion of flesh and tendon, in relative and absolute extent of attachments, the muscles of Chinamen, Frenchmen and negroes are very different. Unfortunately, Chudzinski's methods are open to serious criticism. Take his account of the *soleus* muscle as an example; it is a muscle peculiarly adapted in men for erect progression. He gives in millimetres the extent of its attachment to the fibula: the extent of origin in white men is 135 m.m.: in yellow races, 97 m.m.; in negresses, 94·5 m.m.; in negroes, 120 m.m.; the extent of origin is greatest in white men, as one would expect, but it is obvious that a great error is introduced if no account is taken of the size and stature of the individuals compared, and of this factor Chudzinski has taken no cognisance. Only here and there are relative measurements given. Yet on the whole there can be no doubt that Chudzinski has, in his minute analysis of the muscular system, discovered very numerous features wherein the white, the yellow and the black man differ, and his results may be summarised by saying that, on the whole, the muscular arrangement in the white man is more accurately and powerfully adapted for erect progression of the body and precision in the use of his hands than in the negro, and in some points, although by no means in all, the muscular system is rather more simian in the negro than in the white man.

But by far the most valuable part of Chudzinski's work lies in his observations of muscular anomalies. Adding his work to Giacomini's it may be said, without any danger of over-stating the case, that the negro shows in his muscular anomalies twice or three times the number of simian characters met with amongst white men. Looking at it from the point of view of descent, the negro has retained certain primitive features much more than the white races, among them his black colour.

Little notice was taken in this country of Chudzinski's death, which took place in Paris some eighteen months ago. He was born a Pole, and, when a student at Moscow University, became involved in the insurrection of 1863, and, like many of his countrymen who afterwards became known in science, sought refuge in Paris.